Transport formula for collisional sheet flows with turbulent suspension

Diego Berzi[1]

[1] Assistant professor, Dept. of Environmental, Hydraulic, Infrastructure, and Surveying Engineering, Politecnico di Milano, Milan, 20133, Italy. Email: diego.berzi@polimi.it

Abstract

The prediction of the transport of sediments in streams is of crucial importance for many geophysical and industrial applications. Most of the available formulas for sediment transport are empirical, and apply to situations near initiation, where a few erratic particles are seen jumping and rolling over an immobile bed. Yet, they are commonly adopted for predicting massive transport of sediments, although more rigorous approaches exist. The latter make use of constitutive relations from kinetic theories of granular gases, but require the numerical integrations of complicated, non-linear differential equations; hence, discouraging people from their usage for practical purposes. Here, we propose a new explicit formula for predicting intense sediment transport that is based on kinetic theories of granular gases and incorporates in a simple, yet rigorous, way the possibility of turbulent suspension of the particles. We then show that our formula, unlike others, can quantitatively reproduce physical experiments on steady, uniform flows of natural and artificial particles and water over horizontal, movable beds taken from the literature. Our findings suggest that granular physics is now mature enough to provide practical tools in fields that were so far mainly empirically-oriented.

Introduction and theory

Most of researches on sediment transport has put an emphasis on the forces that the liquid component exerts on the particles, rather than on the particle-particle interactions. This is partially due to the fact that the laboratory experiments were mostly conducted, for practical reasons, at small values of water discharge, close to the inception of particle motion, where inter-particle forces are negligible. At higher values of water discharge, massive transport of sediments takes instead place, and those forces cannot be ignored. Sophisticated mathematical models, that take into account the turbulence of the liquid, the inter-particle collisions, the turbulent suspension and the mutual influence between turbulence and particle velocity fluctuations have been proposed (Jenkins and Hanes 1998; Hsu et al. 2004). They require though the numerical solutions of sets of rather complicated differential equations, for which end-users oriented codes are not available yet; this limits their appeal to audience interested in practical applications.

As recently suggested (Frey and Church 2009), we use up to date findings on granular physics to provide a simple description of intense sediment transport, here defined as the massive flow of particles dominated by collisional exchange of momentum (i.e., Shields numbers higher than about four times the critical value at the inception of particle motion; see later in the text), possibly in presence of turbulent suspension. We focus, for simplicity, on the case of the transport of uniform, rigid spheres of diameter d immersed in water (with ρ and η the water density and viscosity, respectively, and σ the ratio of particle to water density) over a horizontal, plane, movable bed. The shear stress, S^*, exerted by the water at the top of the sediments (actually, at the top of the diffuse collisional layer; see later for more details) represents the driving force of particle motion. It is evident from experiments, that the volume concentration, ν, of the particles increases towards the bed, while both the timeaveraged velocity and the velocity fluctuations of the particles decrease (Sumer et al. 1996; Armanini et al. 2005; Capart and Fraccarollo 2011).

Let us analyze, for the moment, the simpler case of dry granular flows over inclined, movable beds. Particle-particle interactions can be divided into nearly instantaneous collisions and enduring contacts (Berzi et al. 2011), with the former dominant at low to moderate concentrations. For sake of simplicity, but with sufficient accuracy, let us consider that the influence of enduring contacts is almost entirely captured by the presence of a yielding value for the ratio of shear to normal stress (the Coulomb criterion). It is customary to characterize the collisions through a coefficient of restitution, which represents the ratio of pre- to post-collisional relative velocity between two colliding particles, and assume that it is constant (Goldhirsch 2003). Inclined, dry granular flows are characterized by the presence at the top of a ballistic layer, where the mean free path between two consecutive collisions is longer than the ballistic trajectory that every particle follows under the influence of gravity (Pasini and Jenkins 2005). This means that one cannot disregard the influence of external forces – in this case gravity – on the dynamics of particle-particle encounters, and the constitutive relations provided by kinetic theories of granular gases (Jenkins and Savage 1983; Garzo and Dufty 1999) do not apply. When the mean free path diminishes, due to increasing in particle concentration, and is less than the length of the ballistic trajectory, kinetic theories of granular gases are able to provide a correct description of the flow (Goldhirsch 2003). In particular, when the fluctuating velocities of the particles are uncorrelated, classic kinetic theories apply (Garzo and Dufty 1999). Berzi and Jenkins (2011) named this region the diffuse collisional layer. When the concentration further increases – say greater than 0.49 for spheres (Jenkins 2007) – the particle fluctuating velocities are no longer uncorrelated (Kumaran 2009) and one has to modify the classic kinetic theories to account for the diminished energy dissipation in collisions (Mitarai and Nakanishi 2007; Jenkins 2006, 2007). This region is called the dense algebraic layer in Berzi and Jenkins (2011). Below the dense algebraic layer, there is the movable bed, where the ratio of particle shear to normal stress is equal or below the threshold for having motion.

Let us see now how the picture changes in presence of water. At the macroscopic scale, one has to take into account the particle-water interactions (mainly drag, lift and buoyancy) in the momentum equations for the particles, but this does not alter the above mentioned layered structure of the flow. At the microscopic scale, though, the presence of the viscous fluid damps the collisions. Hence, the value of the coefficient of restitution is no longer constant, but is a well defined monotonic function of the particle Stokes number St, i.e., the ratio of particle inertia to fluid viscous forces: it decreases when the Stokes number decreases (Joseph et al. 2001). The subsequent layered structure of sediments is represented in Fig. 1, together with a generic concentration profile. At the top, there is still the presence of a ballistic layer, although in this case the external forces that cannot be disregarded in describing the dynamics of particle-particle encounters are the drag force and the buoyancy, in addition to gravity. Below this, the diffuse collisional and dense algebraic layers are both characterized by the fact that the coefficient of restitution decreases towards the bed, given that the Stokes number is proportional to the square root of the granular temperature, T – the measure of the intensity of particle velocity fluctuations, the analog at the particle scale of the thermodynamic temperature of classic gases – that decreases approaching the bed (Armanini et al. 2005; Berzi 2011). At a certain distance δ from the bed, the Stokes number is so low that the coefficient of restitution vanishes; there, the collisions are perfectly inelastic, and the mixture of sediment and water behaves as a viscous dense suspension, with an effective viscosity that depends on the concentration (Boyer et al. 2011). This is the macro-viscous layer (Berzi 2011), just above the movable bed (Fig. 1). Not all the above described layers are always present in the flow. As shown by Berzi (2011), for values of S^* lower than a certain value, the dense algebraic layer vanishes; on the other hand, for values of S^* greater than another particular value, the macro-viscous layer disappears. Also, we emphasize that the massive transport of sediments is localized in the diffuse collisional, dense algebraic and macro-viscous layers, so that the additional contribution of the ballistic layer is usually negligible. This is not true, though, at the lowest values of S^* (see later) when the massive transport layers vanish; in that case, the sediment transport is concentrated in the ballistic layer.

On the basis of the above described physical picture, we were able to obtain an analytical solution of steady, uniform transport of sediment that applies when the massive transport layers are present, but the particles are not yet suspended by turbulence (Berzi 2011). This means that the total depth h of the massive transport layers, whose expression is reported in Table 1, must be greater than one diameter, and the ratio of the fluid shear velocity at $y = h$, $\left(S^*/\rho\right)^{1/2}$, to the uniform settling velocity of a single particle, w_0, must be less than or equal to one (Jenkins and Hanes 1998). The transport formula – i.e., the particle volume flow rate per unit width, q, as a function of the fluid shear stress S^* – obtained by extending the work of Berzi (2011) to deal with turbulent suspension, in the limit of small values of the coefficient of collisional restitution at $y = H$, i.e., at the top of the dense algebraic layer (Fig. 1) – can be written as:

$$\Phi = \psi\theta_{eff}^{\;3/2}, \tag{1}$$

where $\Phi = q/\left[g\left(\sigma-1\right)\right]^{1/2} d^{3/2}$ is the dimensionless particle volume flow rate per unit width; θ_{eff} is an effective Shields number (see later), which represents a percentage of the actual Shields number, $\theta = S^*/\rho g\left(\sigma-1\right)d$, i.e., the dimensionless fluid shear stress at the top of the massive layers; g is the gravitational acceleration; and the coefficient ψ is an explicit function of θ_{eff} and the set of particle properties (Table 1). The latter includes the coefficient of collisional restitution in absence of water, ε; the parameter c in the expression for the particle velocity correlation in the dense algebraic layer (Jenkins 2006, 2007); the ratio of particle to liquid density, σ; the particle Reynolds number, $\mathrm{R} = \rho d \left[gd \left(\sigma - 1 \right) / \sigma \right]^{1/2} / \eta$, that explicitly depends on the particle diameter; the approximately constant value of the concentration in the dense algebraic and macro-viscous layers, $\bar{\nu}$; and the yielding value of the particle stress ratio at the bed, α. All the particle properties have clear physical meanings and can be easily measured, but for the parameter c, that must be inferred from experiments; nonetheless, c is of order unity and values appropriated for sand, plastic cylinders and glass spheres have been previously determined (Jenkins and Berzi 2010; Berzi 2011). The dependence of ψ on the effective Shields number for natural sand or gravel of different diameters (equivalently, for different value of the particle Reynolds number) in water is plotted in Fig. 2; as in Berzi (2011), we employ the following values for the particle properties: ε=0.45, c=0.65, σ=2.67, $\bar{\nu}$=0.65 and α=0.50. For diameters greater than 10 mm, corresponding to particle Reynolds numbers greater than 2500, the curves collapse onto a single one. It is worth noticing that the most used formulas for sediment transport, such as the famous one proposed by Meyer-Peter and Müller (1948) – MPM formula, from now on –, and its revised form proposed by Wong and Parker (2006) – WP formula –, but also the recent, physically-sounded formula of Capart and Fraccarollo (2011) – CF formula – are characterized by a coefficient of proportionality of Φ with the Shields number to the power of 3/2, independent on both the Shields number, at least far from the inception of particle motion, and the properties of the particles. The literature constant values of the coefficient of proportionality are 8 (MPM formula) and around 4 (WP and CF formulas), while Fig. 2 shows a much wider interval of variation for ψ.

We introduced the effective Shields number in Eq.(1) motivated by the work of McTigue (1981), where a rigorous analysis of the influence of the turbulence on the particles in a mixture resulted in an additional term proportional to the gradient of concentration that must be included in the particle momentum balances (see also Hsu et al. 2004). With this, the particle momentum balance in the vertical direction reads

$$p' = -\nu\rho\left(\sigma - 1\right)g - C\frac{\eta_T}{\rho}\nu' , \tag{2}$$

with p particle pressure, C drag coefficient and η_T turbulent viscosity. The prime indicates the spatial derivative along the vertical direction. We assume that the drag coefficient can be expressed in terms of the uniform settling velocity w of particles at concentration ν, provided that, at equilibrium (i.e., when the drag force is balanced by the buoyant weight),

$$Cw = \nu\rho\left(\sigma - 1\right)g . \tag{3}$$

Both η_T and w are local quantities. However, we can assume that, on average, they are proportional to $\left(\rho S^*\right)^{1/2}\left(h - H\right)$ (McTigue 1981) and w_0, respectively. Then, taking ν to be approximately linearly distributed from 0, at the top, to $\bar{\nu}$, at $y = H$,

$$p' = -\rho\left(\sigma - 1\right)\nu g\left[1 - \bar{\nu}\xi\frac{\left(S^*/\rho\right)^{1/2}}{w_0}\right]. \tag{4}$$

where ξ is a coefficient of order unity. Eq. (4) shows that the collisional pressure decreases when the turbulent suspension is present. The analytical solution resulting in Eq.(1) and Table 1 is based on the determination of the ratio of particle shear stress to particle pressure in the dense layers of Fig. 1 (dense algebraic and macro-viscous layer). There, the dimensionless particle shear stress equals the Shields number, because the turbulence is likely to be suppressed (Berzi 2011). Hence, decreasing the particle pressure by the factor of Eq. (4) is equivalent to increasing the Shields number by the inverse of the same factor. We therefore introduce the effective Shields number as

$$\frac{\theta_{eff}}{1 + \bar{\nu}\xi - \bar{\nu}\xi\left[\theta_{eff}\,g\left(\sigma - 1\right)d\right]^{1/2}/w_0} = \theta . \tag{5}$$

The factor in Eq.(5) has been slightly modified with respect to that in Eq.(4) to ensure that θ_{eff} coincides with θ for $\left(S^{*}/\rho\right)^{1/2}$ equal to w_0 (i.e., when the turbulent suspension vanishes in the diffuse collisional layer).

As already mentioned, the minimum Shields number for which the transport formula (1) holds is that for which the depth of the massive transport layers is at least one diameter. From Table 1, and the idea that at such low values of the Shields number the coefficient of collisional restitution at the top of the dense layers e_H approximately vanishes (Berzi 2011), as well as the turbulent suspension, this implies $\theta > \alpha\bar{\nu}/\left(1.22\alpha+1\right)$. For sand, this minimum value is roughly 0.2, i.e., about four times the critical value for the inception of motion. On the other hand, the maximum value of the actual Shields number for the validity of Eq.(1) is that for which the turbulent suspension is so strong that the collisional pressure at the top of the dense layers vanishes (inception of fully suspended load). Hence, from Eq.(4), $\theta < w_0^{\,2}/\left[\bar{\nu}^2\xi^2 g\left(\sigma-1\right)d\right]$.

Comparison with experiments and conclusions

We now test Eq.(1) against the experimental results reported by Nnadi and Wilson (1992) on the flows of 0.7 mm sand in water (Fig. 3). Those experiments possess some unique features that make them perfect to test sediment transport formulas: (i) they have been performed with natural material, hence satisfying those skeptical about the use of unrealistic artificial particles in laboratory experiments; (ii) they have been obtained on horizontal, plane movable bed, so that additional complications such as gravity in the flow direction and bedforms (Wong and Parker 2006) are ruled out; (iii) the experimentally investigated values of the Shields number, in the interval between 0.8 and 8, pretty much cover the range of most interest for practical applications in Hydraulics (for instance, they are associated with the intense sediment transport occurring during floods).

The turbulent suspension is not expected to play a role for $\theta \leq 1$, given that the single particle, uniform settling velocity for 0.7 mm sand is about 10 cm/s (Abrahams 2003). Indeed, Fig.3 shows that Eq. (1) without turbulent suspension, i.e., with $\theta_{eff} = \theta$, reproduces the experiments up to a value of the Shields number slightly above one, while over-predicts the transport rate for larger values of θ. When we use Eq.(5) to evaluate θ_{eff}, with ξ equal to 0.6, the quantitative agreement of Eq.(1) with the experiments is remarkable (Fig. 3), even at values of θ slightly greater than that for which Eq.(1) is valid (about 6.5 in the case of 0.7 mm sand). Transport formulas other than Eq.(1) constantly under-predict the experimental data (Fig. 3); the better performance of the MPM formula with respect to both WP and CF expressions is likely to be fortuitous, given that the analysis of Wong and Parker (2006) proved that it was based on erroneous interpretation of the experimental results.

Fig. 4 shows the comparison between measured (Nnadi and Wilson 1992) and predicted values of dimensionless particle flow rate per unit width of artificial particles and water. The experiments were performed using mono-dispersed Bakelite beads of two different diameters and water. This allows to investigate the role of the particle Reynolds number on the particle transport rate; with $\sigma = 1.56$, R = 46 and 64, when $d = 0.67$ and 1 mm, respectively. To plot the theoretical curves of Fig. 4, we have employed $\varepsilon=0.6$, $c=0.5$, $\bar{\nu}=0.55$ and $\alpha=0.50$, as appropriated for PVC particles (Berzi 2011); $\xi = 0.6$, as for sand, and w_0 equal to 7 cm/s (Ferguson and Church 2004). As predicted by the theory (Berzi 2011), at lower values of the Reynolds number correspond higher values of Φ, for a given Shields number (Fig.4). Once again, the agreement between the transport formula (1) and the experiments is notable.

We conclude that the proposed formula for predicting intense sediment transport is simple enough to be used for practical purposes, yet rigorous enough, being based on kinetic theories, and granular physics in general, that there is no need for additional tuning parameters; it also has a superior capability of reproducing experiments. The limits of the approach regard, as already mentioned, the prediction of the sediment transport at (i) Shields numbers close to the inception of particle motion (say less than four times the critical value), for which the constitutive relations of kinetic theories do not apply; (ii) Shields numbers larger than the inception of fully suspended sediment transport, at which the collisional pressure vanishes (about 7 for 0.7 mm sand).

Notation

The following symbols are used in the paper:

c = parameter in the expression for the particle velocity correlation;

C = drag coefficient;

d = particle diameter;

e_H = coefficient of collisional restitution at the top of the dense algebraic layer;

g = gravitational acceleration;

h = total depth of the diffuse collisional, dense algebraic and macro-viscous layers;

H = total depth of the dense algebraic and macro-viscous layers;

k = particle stress ratio at the top of the dense algebraic layer;

p = particle pressure;

q = particle volume flow rate per unit width;

R = particle Reynolds number;

S^* = water shear stress at the top of the diffuse collisional layer;

St = Stokes number;

T = granular temperature;

w = uniform settling velocity of particles at concentration ν;

w_0 = uniform settling velocity of a single particle;

x = coordinate in the flow direction;

y = coordinate in the direction perpendicular to the bed;

α = yielding value of the stress ratio at the bed;

δ = depth of the macro-viscous layer;

ε = coefficient of collisional restitution in dry conditions;

Φ = dimensionless particle volume flow rate per unit width;

η = molecular viscosity;

η_T = turbulent viscosity;

λ = particle stress ratio at the top of the macro-viscous layer;

ν = concentration;

$\bar{\nu}$ = approximately constant concentration in the dense layers;

θ = Shields number;

θ_{eff} = effective Shields number;

ρ = water density;

σ = particle density over water density;

ξ = material coefficient;

ψ = coefficient in the transport formula;

ψ_i = auxiliary coefficient (with i = 1, 2, 3).

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Table 1. Summary of the analytical results obtained from the theory of Berzi (2011).

Analytical expression	Definition
$\psi = \psi_1 \theta_{eff}^{1/2} + \psi_2 \theta_{eff} + \psi_3 \theta_{eff}^{3/2}$	Transport coefficient in Eq.(1)
$\psi_1 = \dfrac{69(1+\varepsilon)}{\varepsilon \bar{\nu} \sigma^{3/2} R} \dfrac{(1+e_H)}{(1-e_H/\varepsilon)(0.44+0.61 e_H)} \dfrac{1}{k}$	Auxiliary coefficient
$\psi_2 = \dfrac{0.65 c^{1/2}}{\bar{\nu}^{3/2} \sigma^{1/2}} \dfrac{1}{(0.44+0.3 e_H)^4} \left(\dfrac{\lambda+k}{2}\right)^{9/2} \left(\dfrac{1}{\lambda^{3/2}} - \dfrac{1}{k^{3/2}}\right)$	Auxiliary coefficient
$\psi_3 = 1400 \sigma^{1/2} \bar{\nu} R c^6 \left(\dfrac{\alpha+\lambda}{2}\right)^{18} \left(\dfrac{1}{\alpha^2} - \dfrac{1}{\lambda^2}\right)$	Auxiliary coefficient
$h = \dfrac{\theta_{eff}}{\bar{\nu}} \left(\dfrac{1}{\alpha} + \dfrac{1}{k}\right)$	Total depth of the massive transport layers
$k = 1.24 \left[\dfrac{(0.44+0.61 e_H)(1-e_H)}{1+e_H}\right]^{1/2}$	Ratio of particle shear to normal stress at the top of the algebraic layer
$\lambda = \min\left\{1.87 \left[\dfrac{\bar{\nu}(1+\varepsilon)^2}{c^3 \sigma^2 R^2 \varepsilon^2}\right]^{1/8} \theta_{eff}^{-1/8}; 0.82\right\}$	Ratio of particle shear to normal stress at the top of the macro-viscous layer
$e_H = \varepsilon \max\left\{\dfrac{c^{3/2} \varepsilon \sigma R \theta_{eff}^{1/2} - 27 \bar{\nu}^{1/2}(1+\varepsilon)}{c^{3/2} \varepsilon \sigma R \theta_{eff}^{1/2} + 37 \bar{\nu}^{1/2} \varepsilon (1+\varepsilon)}; 0\right\}$	Coefficient of collisional restitution at the top of the dense algebraic layer

Figure 1
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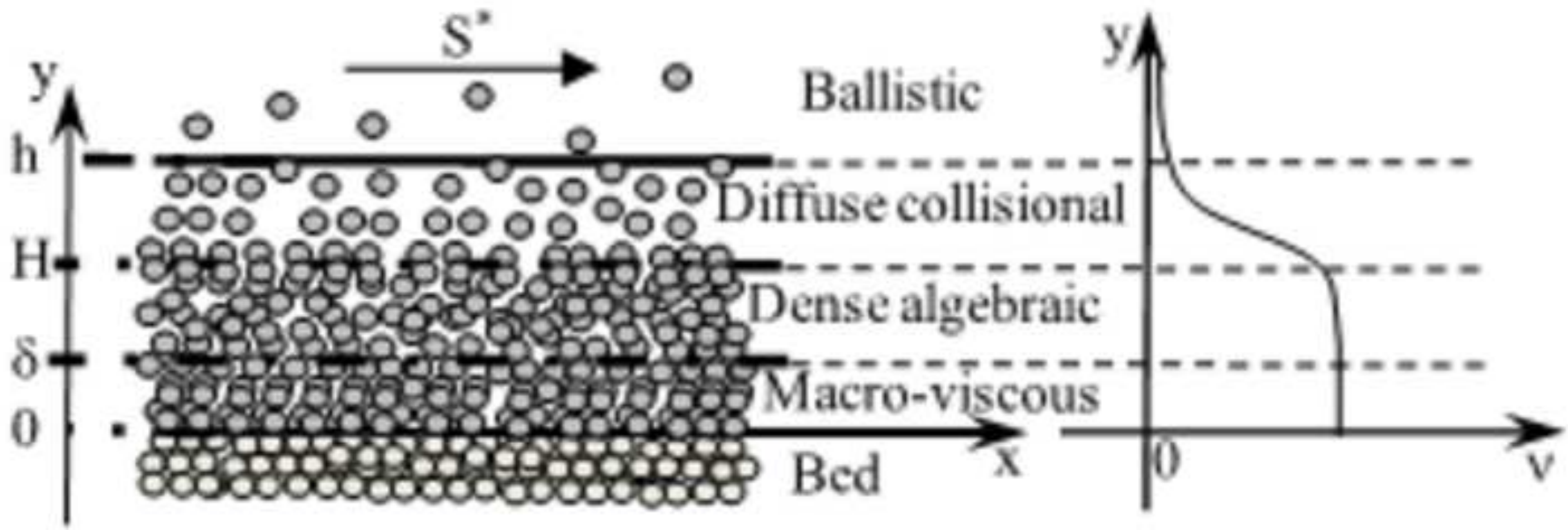

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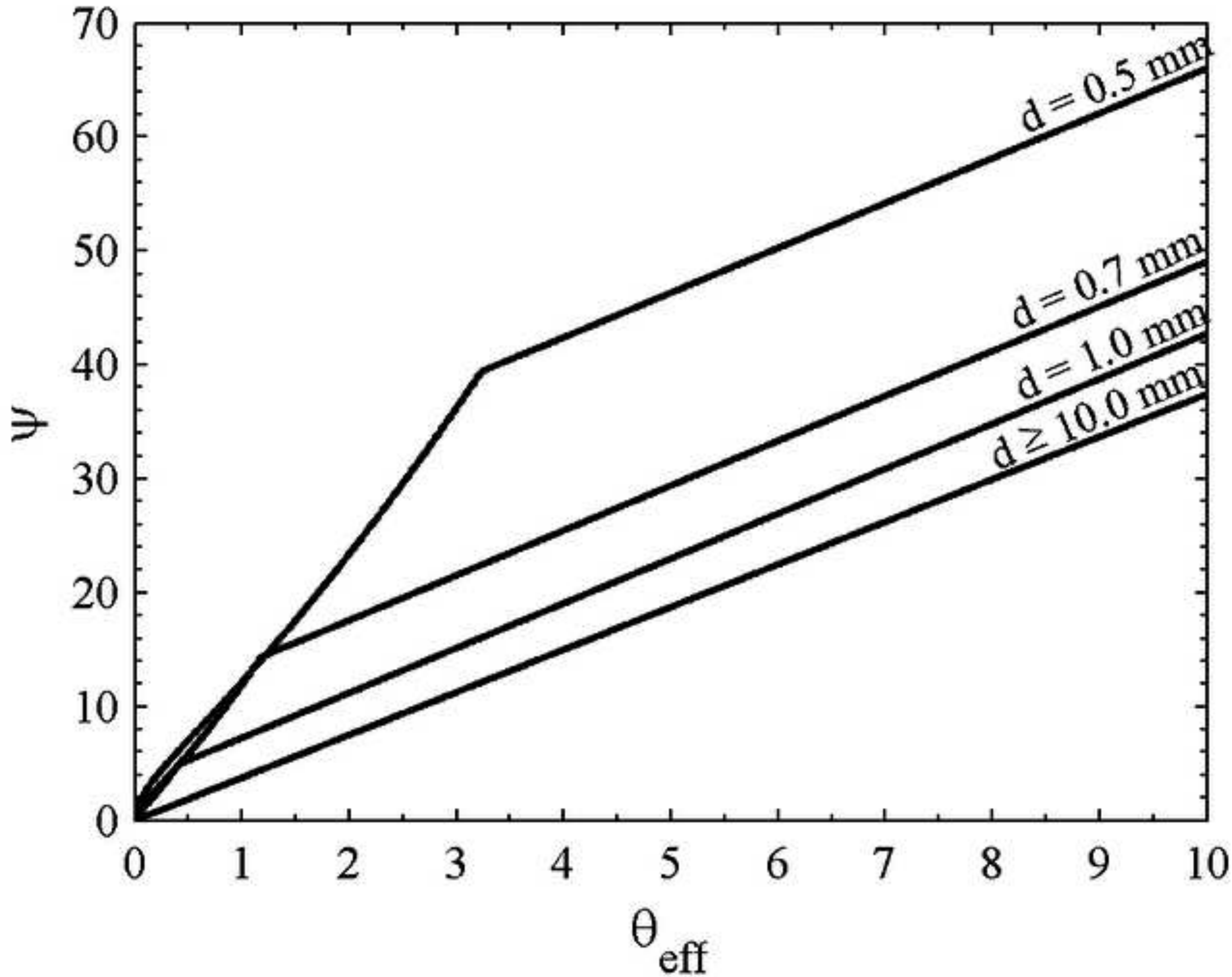

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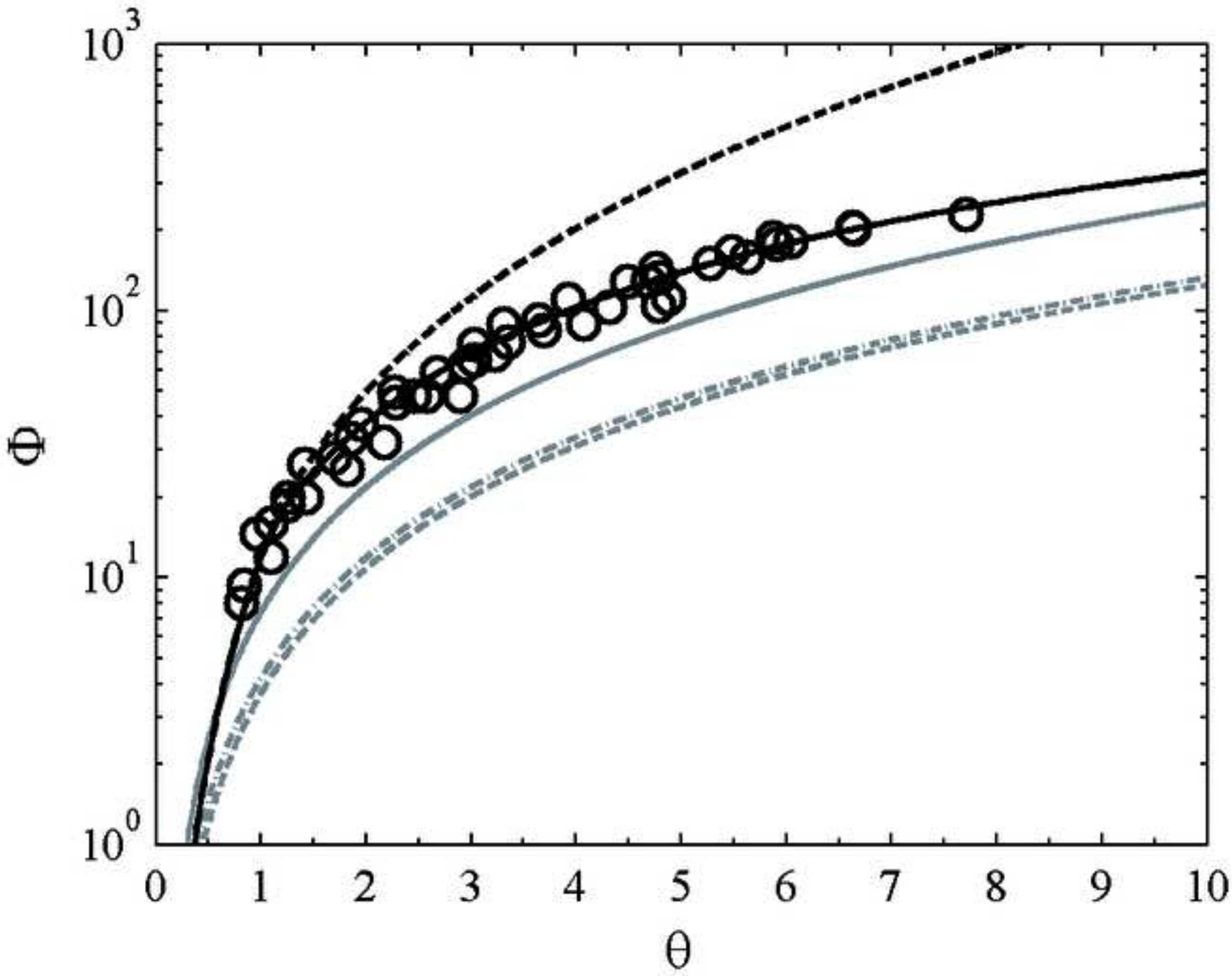

Figure 4

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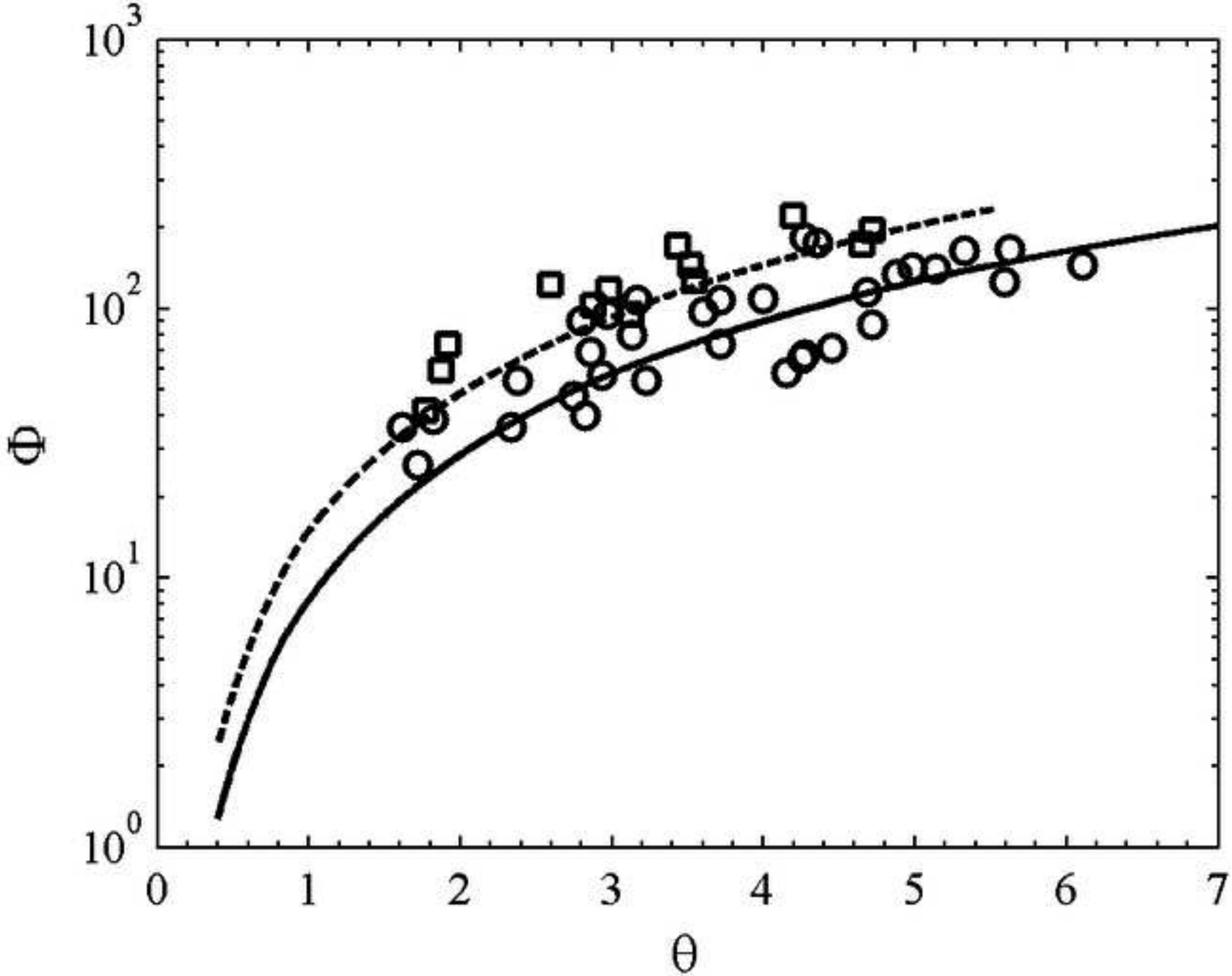

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Figure 1. Layered structure of sediment transport over a horizontal movable bed and associated concentration profile.

Figure 2. Coefficient ψ of Eq.(1) as function of the effective Shields number for sand/gravel of different diameters in water.

Figure 3. Experimental (open circles, after Nnadi and Wilson 1992) data of dimensionless particle volume flow rate per unit width as function of the Shields number for 0.7 mm sand in water. The lines represent the predictions of different formulas for sediment transport: solid black line, Eq.(1); dashed black line, Eq.(1) without including turbulent suspension (i.e., $\theta_{eff} = \theta$); solid gray line, MPM formula; dot-dashed gray line, WP formula; dashed gray line, CF formula.

Figure 4. Experimental (symbols, after Nnadi and Wilson 1992) and theoretical (lines) dimensionless particle volume flow rate per unit width as function of the Shields number for 1 mm (open circles and solid line) and 0.67 mm (open squares and dashed line) Bakelite beads in water.